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Black hole in balance with dark matter

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Equilibrium of a gravitating scalar field inside a black hole compressed to the state of a boson matter, in balance with a longitudinal vector field (dark matter) from outside is considered. Analytical consideration, confirmed numerically, shows that there exist static solutions of Einstein's equations with arbitrary high total mass of a black hole, where the component of the metric tensor $g^{rr}(r)$ changes its sign twice. The balance of the energy-momentum tensors of the scalar field and the longitudinal vector field at the interface ensures the equilibrium of these phases. Considering a gravitating scalar field as an example, the internal structure of a black hole is revealed. Its phase equilibrium with the longitudinal vector field, describing dark matter on the periphery of a galaxy, determines the dependence of the velocity on the plateau of galaxy rotation curves on the mass of a black hole, located in the center of a galaxy.

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The lifetime of a galaxy is of the order of the Universe's life time. So slow evolution suggests that mutual transformations of particles one into another can slow down the collapse of a black hole located in the center of a galaxy, or even stop it. This is the main reason for searching and analyzing static configurations of gravitating objects in general theory of relativity. In the process of gravitational collapse, the density of matter increases continuously. The stage of compression, when massive Bose particles (Z bosons, W bosons, and/or Higgs scalar bosons) are dominating, is inevitable. At low temperatures, the boson matter forms a Bose-Einstein condensate, the wave function of which is that for a scalar field.[1] A possible equilibrium structure of a gravitating scalar field, describing the ordinary matter of a black hole, is considered below. Condition of phase equilibrium at the interface between the scalar field and a longitudinal vector field, describing dark matter outside a black hole, determines the connection (Eq. (3) below) between the plateau velocity of a galaxy rotation curve and the mass of a black hole.

In the framework of the standard approach, a static space-time metric

$$ds^2 = g_{00}(r)\,dx^{02} + g_{rr}(r)\,dr^2 - r^2\left(d\vartheta^2 + \sin^2\vartheta\;d\varphi^2\right),$$

with a spherically symmetric distribution of matter is considered. Using the component of the metric tensor in the form $g_{rr} = -e^{-\lambda}$, possible solutions with g_{rr} changing sign are excluded in advance. Solutions, with the sign of g_{rr} remaining unchanged, exist, but only if the total mass M does not exceed a critical value M_{cr}. To leave a possibility for g^{rr} to change sign, the Einstein equations (100.4) and (100.5) in Ref. 2 should be written as

$$(g^{rr})' + \frac{1 + g^{rr}}{r} = \kappa r T_0^0, \tag{1}$$

$$g^{rr}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) + \frac{1}{r^2} = \kappa T_r^r. \tag{2}$$

Gravitational constant $\kappa = (8\pi k)/c^4 = 2\times 10^{-48}\,\mathrm{sec}^2/\mathrm{g}\times\mathrm{cm}$, $k = 6.67\times 10^{-8}\mathrm{cm}^3/\mathrm{g}\times\mathrm{sec}^2$, $\nu = \ln g_{00}$.

Looking at Eq. (1) one can see, that in the region where $g^{rr} > 0$, the derivative $(g^{rr})'$ becomes negative when $\frac{1+g^{rr}}{r}$ exceeds $\kappa r T_0^0$ with growing r. It means, that there may exist (and do exist) a solution where, with increasing r, the metric component $g^{rr}(r)$ gets into the region of violated signature of metric tensor, passes through a maximum, and returns back into the region of the Galilean signature. $g^{rr}(r)$ intersects the x axis twice. Note, that the component g^{00} of the metric tensor does not change sign. It does not lead to any logical contradiction, provided that r_h is the event horizon, unattainable for a remote observer. Suppose, the metric signature is changed to $(+,+,-,-)$ within a spherical layer $r_g < r < r_h$. Then $g^{rr}(r_g) = g^{rr}(r_h) = 0$.

It follows from the Einstein equations (1), (2) that

$$T_0^0(r_h) = 0, \quad T_r^r(r_h) = \frac{1}{\kappa r_h^2}.$$

The energy density T_0^0 is zero at the horizon $r = r_h$. However, since $T_r^r(r_h) \neq 0$, the pressure on the surface does not disappear. It means that there can be no static equilibrium state of a gravitating body **in vacuum**, if the signature of metric tensor is broken. Nevertheless, one can not exclude a possibility of static equilibrium, if the event horizon r_h takes place on the interface between two gravitating objects. For instance, if the pressure on the surface of a black hole from inside is compensated by the pressure of dark matter from outside. The balance of pressures at the interface of black hole and dark matter establishes a relationship between the parameters of these phases. In particular, we find the dependence of the plateau velocity V_{pl} of a galaxy rotation curve on the mass M of a black hole:

$$V_{\mathrm{pl}} = c\frac{M_{\mathrm{Pl}}^2}{4\sqrt{\mu m}M}, \tag{3}$$

$M_{\rm Pl} = \sqrt{\hbar c/k} = 2.177 \times 10^{-5}$g is the Plank mass; m is the mass of a quantum of scalar field; μ is the mass of a quantum of longitudinal vector field (dark matter).

Time is a cyclic coordinate in a static field. The energy of a single quantum $E = \hbar\omega$ is the integral of motion. A scalar field in the state of definite energy E per particle has the form: $\psi_E\left(x^i\right) = e^{-iEx^0/\hbar c}\psi\left(r\right)$ .Radial function $\psi\left(r\right)$ obeys the Klein-Gordon equation

$$g^{rr}\psi'' + \left(\left(g^{rr}\right)' + \frac{g' g^{rr}}{2g}\right)\psi' = \frac{1}{\hbar^2 c^2}\left(g^{00}E^2 - m^2c^4\right)\psi, \quad g = \det g_{ik}. \tag{4}$$

Note that g^{rr} is the coefficient at the highest derivative in Eq. (4). Since $g^{rr}\left(r_h\right) = 0$, the Klein-Gordon equation is not determined on the hypersphere $r = r_h$.

It is convenient to present the set of the Klein-Gordon and Einstein equations (4), (1)–(2) in the normal form. In the dimensionless variables $x = \frac{mc}{\hbar}r$, $u\left(x\right) = \sqrt{\kappa}\psi\left(r\right)$, $w\left(x\right) = \frac{\hbar\sqrt{\kappa}}{mc}g^{rr}\left(r\right)\psi'\left(r\right)$, $g\left(x\right) = g^{rr}\left(r\right)$, $h\left(x\right) = \frac{E^2}{m^2c^4}g^{00}\left(r\right)$ we have the system of four first order equations, resolved with respect to derivatives:

$$u' = w/g, \tag{5}$$

$$w' = \left(h-1\right)u - \frac{2w}{x} + \frac{x}{g}\left(u^2 h - \frac{w^2}{g}\right)w, \tag{6}$$

$$g' = x\left(\left(1+h\right)u^2 - \frac{w^2}{g}\right) - \frac{1+g}{x}, \tag{7}$$

$$h' = \frac{h}{x}\left(1 + \frac{1}{g}\right) - \frac{xh}{g}\left(u^2\left(1-h\right) + \frac{w^2}{g}\right). \tag{8}$$

A detailed derivation of these equations is presented in the article,[3] devoted to equilibrium states of a gravitating boson condensate with g^{rr} not changing sign.

The set (5) – (8) contains no parameters. Here and below, $g(x)$ is the component $g^{rr}\left(r\right)$ of the metric tensor. Please, do not confuse it with the determinant $g = \det g_{ik}$ in the Klein-Gordon equation (4). Denote a dimensionless gravitational radius $x_g = \frac{mc}{\hbar}r_g$. At $x = x_g$ the component of metric tensor $g(x_g) \equiv g^{rr}(r_g) = 0$.

It follows from Eq. (8), that at $x \to x_g \pm 0$ ($+0$ means from above, and -0 - from below)

$$\left(\frac{w^2}{g}\right)_{x\to x_g \pm 0} \to \frac{1}{x_g^2} - u_{g\pm}^2\left(1 - h_{g\pm}\right). \tag{9}$$

Here $u_{g\pm} = \lim\limits_{x\to x_g\pm 0} u(x)$, $h_{g\pm} = \lim\limits_{x\to x_g\pm 0} h(x)$ are one-sided limits either from above, or from below. Substituting (9) into Eq. (7), we obtain:

$$g'\left(x_g\right) = \frac{2}{x_g}\left(x_g^2 u_g^2 - 1\right). \tag{10}$$

It follows from (9) and (10) that

$$w^2(x) = \left[x_g^{-2} - u_g^2(1 - h_{g\pm})\right] g'(x_g)(x - x_g) \quad \text{at } x \to x_g \pm 0. \tag{11}$$

On the left-hand side $w^2 > 0$. Hence, the right-hand side of (11) should also be positive. The factor $(x - x_g)$ changes sign at $x = x_g$. Therefore, the combination $x_g^{-2} - u_g^2(1 - h_{g\pm})$ has different signs at $x < x_g$ and $x > x_g$. There are two possibilities:

1. Regular gravitational radius. In the case of exact connection between the parameters $h_{g-} = h_{g+} \equiv h_g = 1 - (x_g u_g)^{-2}$ all four functions, $u(x)$, $w(x)$, $g(x)$, $h(x)$, as well as their derivatives, are continuous at $x = x_g$. In accordance with (9) $w^2/g = 0$ at $x = x_g$. x_g is a regular gravitational radius inside the gravitating condensate, provided that its energy-momentum tensor is regular.

2. Termination point. If $g(x_h) = 0$, and $h_h \neq 1 - (x_h u_h)^{-2}$, the solution cannot be continuously extended to the area $x > x_h$. Functions $u(x)$, $w(x)$, and $h(x)$ terminate at $x = x_h$ in a square root manner, while $g(x)$ remains a linear function. Termination point x_h is the event horizon for a remote observer.

Analytical analysis, confirmed numerically, shows, that within the strip $1 < x_g^2 u_g^2 < 2$, $x_g < x_{g\,\max} = 1.4285$ on the plane of parameters x_g, u_g (Fig. 1(A)) there are finite-mass solutions, continuous at the regular gravitational radius $x = x_g$, and terminating at the horizon $x = x_h > x_g$ in a square root manner. Approaching the upper line in Fig. 1(A) the mass grows, and becomes infinite on the line.

An example of a regular solution with parameters $x_g = 1, u_g = \sqrt{2} - 0.01$ is shown in Fig. 1(B). In the interval $1 < x < 13.78585$ the metric component $g(x) > 0$. This is the region of changed signature. On the right boundary $x_h = 13.78584985$ the scalar field $u(x)$ terminates with a nonzero value. On enlarged scale, this solution is shown in Fig. 1(C) in the vicinity of the horizon x_h.

The main factor allowing the existence of supper-heavy black holes, and providing the ability to stop the collapse, is the presence of a layer of broken signature (where $g^{rr} > 0$) inside a black hole. Supermassive black holes in the centers of galaxies, being in the state of phase equilibrium with the surrounding dark matter, form the skeleton of modern structure of the Universe.

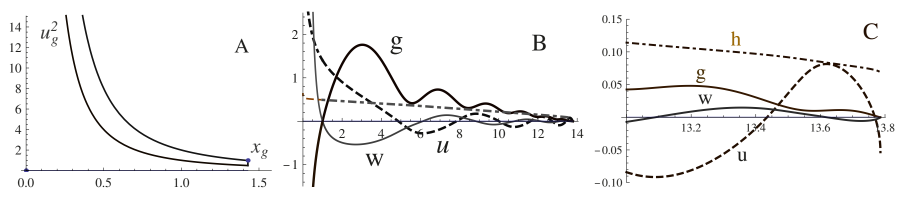

Fig. 1. A - The strip in the plane of parameters x_g, u_g^2 between lower $u_g^2 = x_g^{-2}$ and upper $u_g^2 = 2x_g^{-2}$ lines is the area of existence of regular solutions with a finite mass M. B - An example of a regular solution. $x_g = 1, u_g = \sqrt{2} - 0.01$. The horizon $x_h = 13.78581985$ is found numerically. C - The same functions $g(x), u(x), h(x), w(x)$ in the vicinity of the horizon x_h.

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