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Interpretation of High-Dimensional Numerical Results for the Anderson Transition[1]

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Abstract—The existence of the upper critical dimension $d_{c2} = 4$ for the Anderson transition is a rigorous consequence of the Bogoliubov theorem on renormalizability of ϕ^4 theory. For $d \geq 4$ dimensions, one-parameter scaling does not hold and all existent numerical data should be reinterpreted. These data are exhausted by the results for $d = 4, 5$ from scaling in quasi-one-dimensional systems and the results for $d = 4, 5, 6$ from level statistics. All these data are compatible with the theoretical scaling dependences obtained from Vollhardt and Wolfle's self-consistent theory of localization. The widespread viewpoint that $d_{c2} = \infty$ is critically discussed.

Contribution for the JETP special issue in honor of A.F. Andreev's 75th birthday

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1. INTRODUCTION

The main defect in the current literature on the Anderson transition is the ignorance of the upper critical dimension $d_{c2} = 4$, which is a rigorous consequence of the Bogoliubov theorem on renormalizability of ϕ^4 theory [1, 2]. The problem of the Anderson transition can be reduced (in a mathematically exact manner) to one of the variants of ϕ^4 theory [3–6],[2] which is nonrenormalizable for space dimensions $d > 4$. Therefore, the cutoff momentum Λ, corresponding to the atomic length scale a, cannot be excluded from the results. It rules out the existence of one-parameter scaling [9], according to which the correlation radius ξ is the only essential length scale. In the latter case, any dimensionless quantity Q related to a finite system of size L can be written as a function of ratio L/ξ,

$$Q = F(L/\xi), \tag{1}$$

which is the basis for all numerical algorithms. The study of Q as a function of L and distance to the transition τ makes it possible to determine the critical exponent ν of the correlation length ($\xi \sim |\tau|^{-\nu}$). Indeed, if two L-dependences for $\tau = \tau_1$ and $\tau = \tau_2$ are calculated, then the scale transformation makes it possible to determine the ratio of two correlation lengths. Persisting this procedure for the sequence τ_1, τ_2, τ_3, ..., one can determine dependence $\xi(\tau)$ apart from the numerical factor.

Relation (1) is invalid for $d > 4$, and a more general form should be used:

$$Q = F(L/\xi, L/a). \tag{2}$$

For $d = 4$, the situation is more complicated and needs additional study. In fact, the existence of logarithmic factors like $\ln(L/a)$ leads to a relation of type (2). The latter can be reduced to a function of one argument if an appropriate choice of the scaling variables is made [10, 11].

Currently, the following results exist for the Anderson transition in high dimensions: the data by Markos for $d = 4$, $d = 5$ [12, 13], obtained from scaling in quasi-one-dimensional systems; the data by Zharekeshev and Kramer for $d = 4$ [14], and those by Garcia-Garcia and Cuevas for $d = 5$, $d = 6$ [15], obtained from level statistics. All these results are based on relation (1) and need reinterpretation due to the above arguments.

In Sections 3 and 4, these results are compared with the modified scaling for high dimensions obtained in [10, 11] from Vollhardt and Wolfle's self-consistent theory of localization [17]. The latter gives correct values of the upper critical dimension $d_{c2} = 4$ and the exponent $\nu = 1/2$ for $d > d_{c2}$, and at least is of interest as a possible scenario. According to certain arguments [18, 19], Vollhardt and Wolfle's theory predicts the exact critical behavior and a lot of the numerical results can be matched with it [10, 11, 20, 21]. This paper supports the same tendency: all indicated numerical data [13–15] can be matched with the the-

[1] The article was translated by the author.

[2] Specifically, to the problem of two zero-component interacting fields [4, 5]. The arguments on the deficiency of the replica method [7, 8] are insignificant in the given context, since renormalizability can be analyzed at the diagrammatic level. Each diagram of disordered systems theory can be obtained from a certain diagram for ϕ_4 theory by simple replacement of symbols [3].

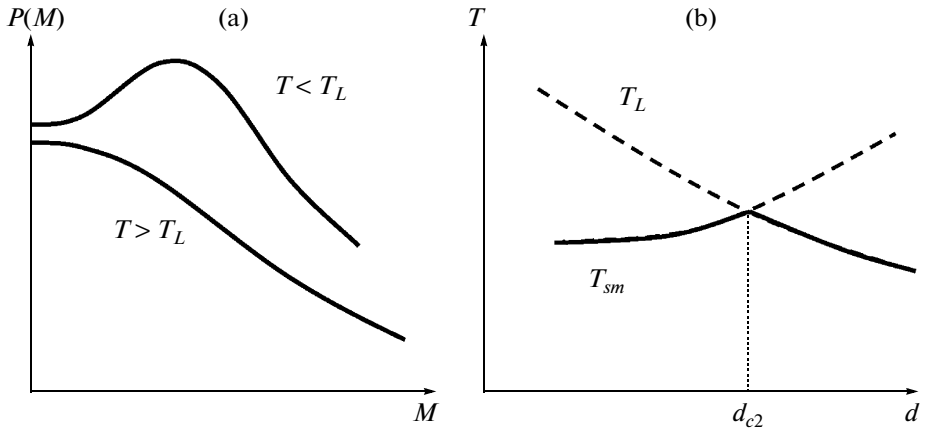

Fig. 1. (a) The sigma-model concept can be used only in the case when the distribution function $P(M)$ of the modulus of M has a maximum at finite M. Such a maximum occurs as a result of the mean-field type transition in the spirit of the Landau theory. (b) Temperature T_L at which a sigma-model occurs and critical point T_{sm} according to the sigma-model scenario as functions of space dimensionality d. Solid and dashed lines show the real and fictitious phase transitions.

oretical scaling dependences. As a rule, the "experimental" points lie on quasi-linear portions of the scaling curves; this was interpreted as the dependence $L^{1/\nu}$ with $\nu \approx 1$ in the original papers. The actual critical behavior suggests $\nu = 1/2$, but the corresponding parts of the scaling dependences are difficult to study in numerical experiments due to their limited accuracy.

The widespread viewpoint that $d_{c2} = \infty$ [22–27] is discussed in the next section.

2. SIGMA-MODELS AND d_{c2}

The hypothesis that $d_{c2} = \infty$ is based on the following arguments:

(a) In the approach based on the use of sigma-models [16], there are no indications to the existence of special dimensionality in the interval $2 < d < \infty$ [22].

(b) Results $s = \infty$, $\nu = 1/2$ for $d = \infty$ [23, 24] (s is the critical exponent of conductivity) demonstrate the validity of the Wegner relation $s = \nu(d-2)$ for $d = \infty$, and one can expect that this relation (and, consequently, the one-parameter scaling picture [9]) is valid for all $d > 2$.

We do not question the results $s = \infty$, $\nu = 1/2$ for the infinite-dimensional sigma-model, but there is the problem of their correspondence with the initial disordered system. For electrons in the random potential, derivation of sigma-models is substantiated only for dimensions $d = 2 + \epsilon$ with $\epsilon \ll 1$; qualitatively, it can be extended to $\epsilon \sim 1$ but not to $d \gg 1$. Extension of sigma-models to higher dimensions is based on an artificial construction corresponding to a system of weakly connected metallic granules [22].

In each granule, only the zero Fourier component of the matrix field Q is taken into account, while connections between granules are supposed to produce only slow variation of Q. The possibility that a coupling between granules leads to induction of higher Fourier components and almost complete destruction of the sigma-model is not considered, while such a situation looks rather probable from the standpoint of spatially homogeneous systems.[3]

Let us explain the situation using the vector sigma-model as an example. If a ferromagnet is described in terms of ϕ^4 theory, then the magnetic moment $\mathbf{M}$ of a finite block can be considered as the Heisenberg spin with a certain fluctuation of its modulus M. Neglecting such longitudinal fluctuations (which can be rigorously justified in dimensions $d = 2 + \epsilon$) by definition corresponds to a sigma-model. For $d = 3$, the longitudinal fluctuations have no qualitative effect and the sigma-model seems applicable.[4] However, when d approaches to 4, the longitudinal fluctuations become anomalously soft and this is the origin of the upper critical dimension. If longitudinal fluctuations are artificially suppressed (which is the case in sigma-models), then it can lead to elimination of d_{c2}.

In fact, one can use the sigma-model concept only in the case when the distribution function $P(M)$ of modulus of $\mathbf{M}$ has a maximum at finite M (Fig. 1a). With a decrease in temperature, such a maximum arises as a result of the mean-field-type transition in the spirit of the Landau theory. However, the corre-

[3] In our opinion, it is practically evident. If field Q is indeed slowly varying, then it remains almost constant inside the block composed of several granules; it means validity of the Wigner–Dyson statistics for this block [28]. In fact, for a coupling strength corresponding to the Anderson transition, hybridization of the block eigenfunctions is not complete (i.e. with equal weights, like in the metallic phase) but partial (which is typical of the critical region). Hence, the level statistics for the composed system of several granules will significantly differ from the Wigner–Dyson one.

[4] In fact, it is possible to show (see [29], Section 3.1) that equivalence of the sigma-model and ϕ^4 theory in the sense of the critical behavior takes place for $d < 4$.

sponding temperature T_L does not signify the actual phase transition, since the transverse fluctuations of **M** destroy the long-range order. Long-range correlations for transversal fluctuations arise at a lower temperature T_{sm}. Therefore, with a decrease in temperature, first (at point T_L), a sigma-model arises, and secondly (at point T_{sm}) the phase transition takes place according to the sigma-model scenario. One can imagine that with a change in d, dependences $T_L(d)$ and $T_{sm}(d)$ intersect at point d_{c2} (Fig. 1b). Hence, for $d > d_{c2}$, the phase transition occurs at the same point T_L where a sigma-model occurs, i.e., according to the Landau scenario.

One can see that the sigma-model transition (at point T_{sm}) is not achieved for $d > d_{c2}$, if the properties of the sigma-model correspond to the properties of the real system. However, such transition can exist (as a theoretical construction) and correspond to the analytical continuation from lower dimensions, if a sigma-model is introduced artificially. In our opinion, exactly such situation takes place in the Anderson transition theory: the results $s = \infty$, $\nu = 1/2$ for $d = \infty$ correspond to the formal sigma-model and not to the initial disordered system.[5] Direct analysis of the Bete lattice (without use of sigma-models) yields $s = 1$, $\nu = 1/2$ [30, 31], in correspondence with Vollhardt and Wolfle's theory.

In addition, let us discuss the paper [27], where the conclusion that $d_{c2} = \infty$ is drawn from a modification of the self-consistent theory. This paper is based on the erroneous idea that the L-dependence for the diffusion constant in the critical point $D_L \sim L^{2-d}$ signifies the existence of the momentum dependence $D(q) \sim q^{d-2}$. In fact, the L-dependence arises not only from spatial, but also from temporal dispersion, and for a given function $D(\omega, q)$, it is determined by the relation

$$D_L \sim D(D_L/L^2, L^{-1}). \tag{3}$$

If a power law dependence on ω and q is accepted, then the combination

$$D(\omega, q) \sim \omega^{\eta/d} q^{d-2-\eta} \tag{4}$$

ensures the correct behavior $D_L \sim L^{2-d}$ for any value of exponent η [32]. If one repeats the construction of [27] with an arbitrary value of η, then it is easy to test that the Wegner relation $s = \nu(d-2)$ is valid (for $d < d_{c2}$) only for $\eta = d - 2$, i.e., in the absence of spatial dispersion.[6] For the choice $\eta = 0$ made in [27], the Wegner relation is violated and the main argument of this paper (agreement with numerics) becomes fictitious, since all numerical results are based on the one-parameter scaling [9].

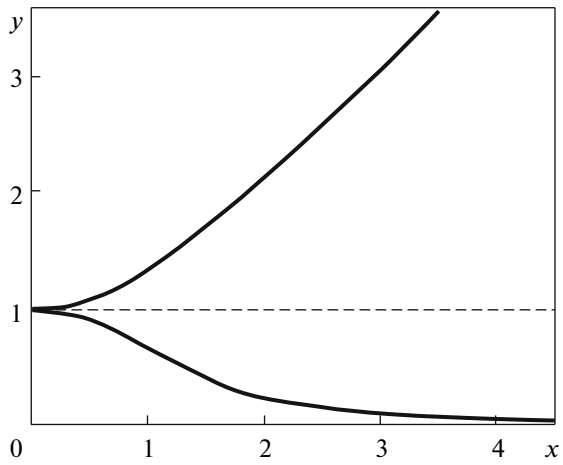

Fig. 2. Scaling function $y(x)$ for an algorithm based on the use of auxiliary quasi-one-dimensional systems, corresponding to high dimensions.

3. QUASI-ONE-DIMENSIONAL SYSTEMS

One of the popular numerical algorithms is based on consideration of auxiliary quasi-one-dimensional systems [34], whose correlation length ξ_{1D} is always finite. As a scaling parameter one can use the quantity [10, 12]:

$$z_1 = L/\xi_{1D}, \tag{5}$$

where ξ_{1D} is the correlation radius of a quasi-one-dimensional system. In numerical experiments, ξ_{1D} is estimated as the inverse of the minimal Lyapunov exponent [34, 35], while a scaling relation of type (1) is postulated for z_1. In high dimensions, such a relation is invalid and one should use the modified scaling suggested in [10]. The theoretical scaling function $y(x)$ is determined by the equation

$$\pm x^2 = \frac{1}{y} - y^2, \tag{6}$$

where variables y are x are defined as

$$y = \frac{L}{\xi_{1D}}\left(\frac{L}{a}\right)^{(d-4)/3}, \quad x = \frac{L}{\xi}\left(\frac{L}{a}\right)^{(d-4)/3} \tag{7}$$

for $d > 4$, and

$$y = \frac{L}{\xi_{1D}}\left[\ln\left(\frac{L}{a}\right)\right]^{1/3}, \quad x = \frac{L}{\xi}\frac{[\ln(\xi/a)]^{1/2}}{[\ln(\xi/a)]^{1/6}} \tag{8}$$

for $d = 4$. Dependence $y(x)$ consists of two branches and is shown in Fig. 2. It is clear from (6)–(8) that the usual scaling constructions are possible if quantity y is considered as a function of the "modified length" $\mu(L) = L^{(d-1)/3}$ $(d > 4)$ or $\mu(L) = L[\ln(L/a)]^{-1/6}$ $(d = 4)$.

[5] According to [19], the high-dimensional sigma-model is unstable to small perturbations of the general form, as a consequence of the unusual result $\epsilon(0, 0) \sim \xi$ [23] for the dielectric constant $\epsilon(\omega, q)$; its difference from the natural result $\epsilon(0, 0) \sim \xi^2$ indicates the existence of the special dimension. It should be noted that the physical sense of the upper critical dimension $d_{c2} = 4$ can be completely clarified for the problem of the density of states [6].

[6] Absence of spatial dispersion $D(\omega, q)$ is obtained in [19] by a detailed analysis. Arguments relating the exponent η with multifractality of wavefunctions [32] are logically defective [33].

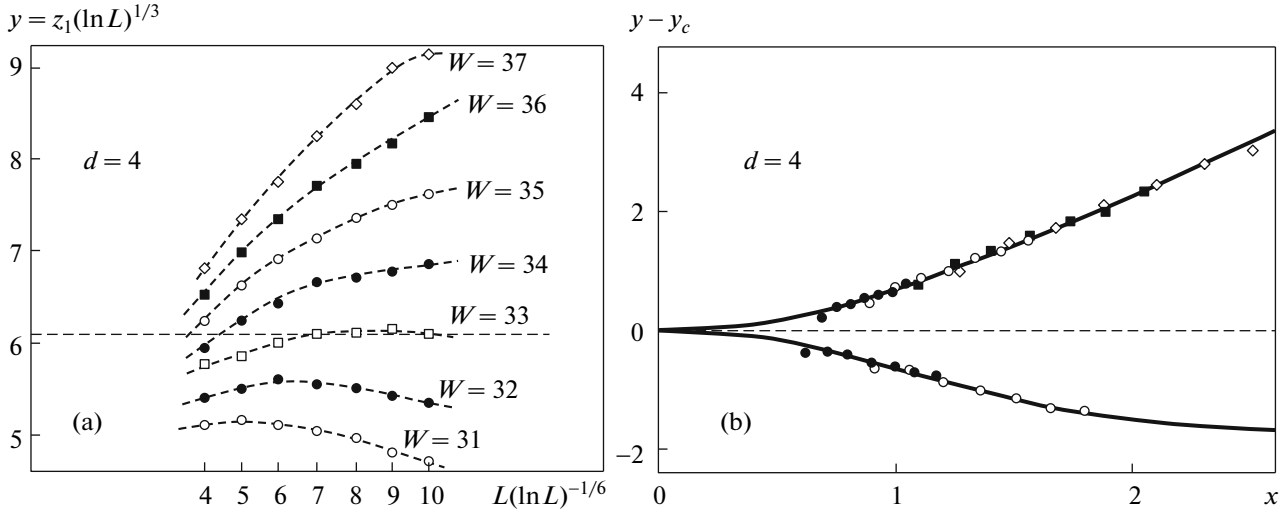

Fig. 3. (a) Numerical data by Markos for $d = 4$ (quasi-one-dimensional systems) extracted from Fig. 61 of [12]; figures near the horizontal axis show the corresponding value of L. (b) Their comparison with the theoretical scaling dependence.

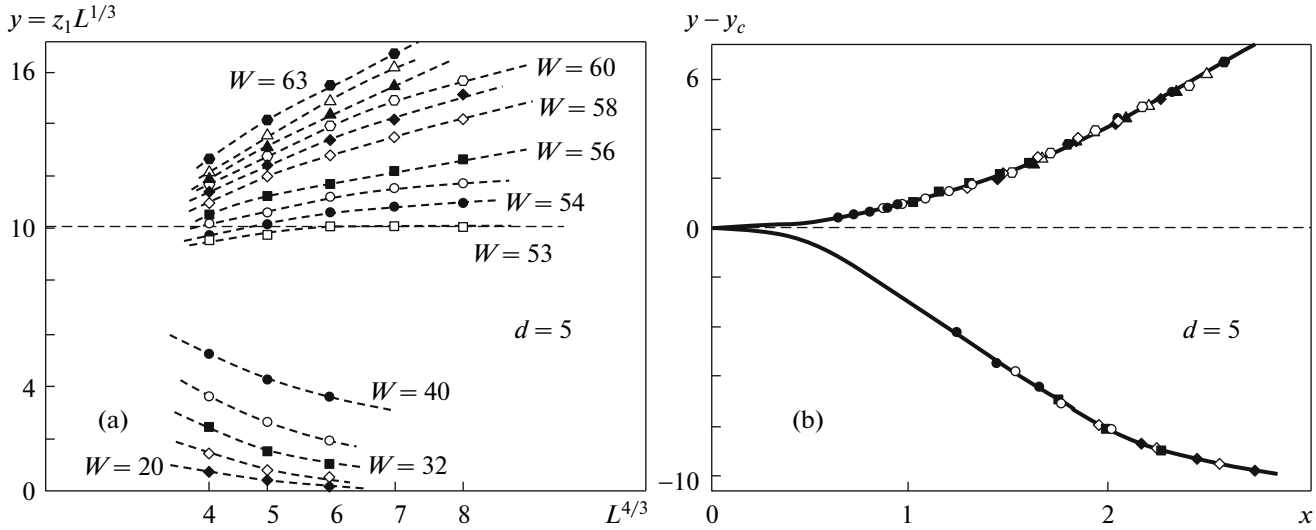

Fig. 4. Numerical data by Markos for $d = 5$ (quasi-one-dimensional systems) extracted from Fig. 61 of [12] and Figs. 4, 5 of [13] (a); their comparison with the theoretical scaling dependence (b).

Figure 3a illustrates the numerical data by Markos for $d = 4$ extracted from Fig. 61 of [12] and represented as $z_1 (\ln L)^{1/3}$ versus $\mu(L)$. The constant limit is achieved for $W = 33$, which gives an estimate of the critical point somewhat different from $W_c = 34.3$ in [12]. Accepting $y(W, L) - y(33, L)$ as $y - y_c$, one can put all numerical data on the theoretical scaling curve by changing the scale along the horizontal axis (Fig. 3b) if the common scale along the y axis is chosen in the appropriate manner.

Figure 4a shows numerical data by Markos for $d = 5$ extracted from Fig. 61 of [12] and Figs. 4 and 5 of [13], represented as $z_1 L^{1/3}$ versus $L^{4/3}$. The change of the treatment procedure led to a significant shift of the critical point, from $W_c = 57.3$ [12] to $W_c = 53$, and made it close to the estimate $W_c = 51.4$ of [15] obtained from level statistics (see Section 4). Accepting $y(W, L) - y(53, L)$ as $y - y_c$, one can put all experimental points on the theoretical scaling curve (Fig. 4b).

In both cases, the main body of data lies on quasi-linear portions $y \sim x$ of the scaling curves and corresponds to dependences $z_1 \sim L(\ln L)^{1/3}$ for $d = 4$ and $z_1 \sim L$ for $d = 5$, which were interpreted in [12] as $z_1 \sim L^{1/\nu}$ with $\nu \approx 1$. In fact, these data are consistent with the predictions of Vollhardt and Wolfle's theory, which gives $\nu = 1/2$; the corresponding dependence $y - y_c \sim x^2$ is valid only for small deviations from the critical

point, which are comparable with the scattering of experimental points.

To conclude the section, let us discuss the technical moment related to the choice of scaling procedure. The scaling constructions can be carried out in usual or logarithmic coordinates, which is absolutely identical in the case of rigorous scaling. In actuality, logarithmic scaling may not yield sufficiently smooth matching of two "pieces" of the measured dependence, since it rigidly fixes the origin of the L axis. Scaling in usual coordinates allows a more smooth matching of pieces due to small shifts along the horizontal axis. Such shifts should be absent in the case of exact scaling, but in practice they arise due to scaling corrections. For example, the structure of scaling corrections to relation (1) has the following form for small τ [10]:

$$y - y_c = \tau\{A_0 L^{1/\nu} + A_1 L^{\omega_1} + A_2 L^{\omega_2} + \ldots\} + \{B_1 L^{-\alpha_1} + B_2 L^{-\alpha_2} + \ldots\}, \quad (9)$$

where $1/\nu > \omega_1 > \omega_2 > \ldots$, $\alpha_1 < \alpha_2 < \ldots$. In the accepted interpretation of $y - y_c$ as $y(W, L) - y(W_c, L)$, the term in the second pair of brackets is excluded from consideration. The expression in the first pair of brackets is dominated by $L^{1/\nu}$ for large L, while other terms are dominant for small L: effectively, it shifts the origin of the L axis. With variable L/ξ used instead of L, such a shift becomes τ-dependent. If the numerical data are sufficiently detailed to provide matching of pieces from the smoothness condition, then such a procedure makes it possible to account for the main scaling corrections. For this reason, we use the usual and not logarithmic coordinates.

4. LEVEL STATISTICS

In the analysis of level statistics [11], the following combination is of the main interest:

$$y = \sigma^2/\sigma_P^2, \quad (10)$$

where σ is the root-mean-square fluctuation of the number of levels N in the energy interval $E = s\Delta$, where Δ is the mean level spacing in a finite system, and σ_P is a value of σ for the Poisson statistics. This quantity is closely related to parameter A in the asymptotics of the distribution function $P(s)$

$$P(s) \sim \exp(-As), \quad A = \sigma_P^2/\sigma^2 \quad (11)$$

of the distance $\omega = s\Delta$ between nearest levels in the large s limit. According to [11], quantity (10) is a function of variable x, which is defined as

$$x = s^{-1/4}\frac{L}{\xi}\left(\frac{L}{a}\right)^{(d-4)/4}, \quad d > 4, \quad (12)$$

$$x = s^{-1/4}\frac{L}{\xi}\frac{[\ln(\xi/a)]^{1/2}}{[\ln(L/a)]^{1/4}}, \quad d = 4. \quad (13)$$

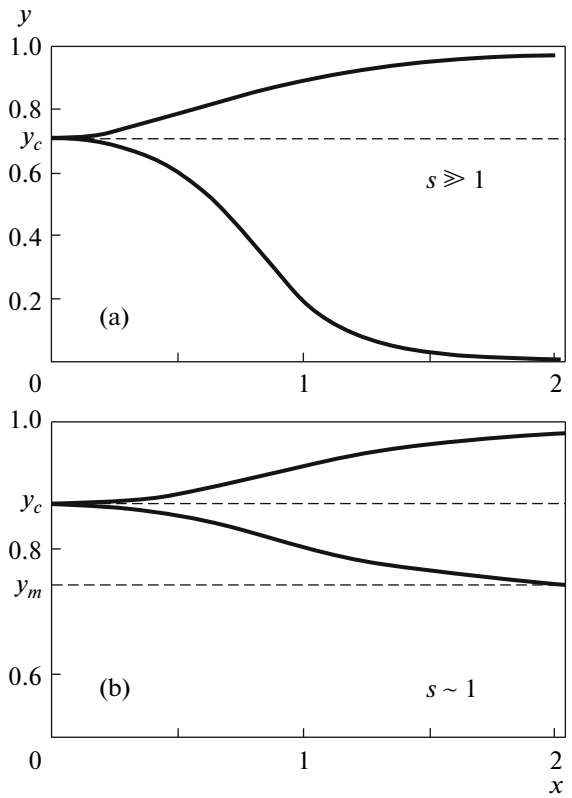

Fig. 5. Examples of scaling dependences $y(x)$ for level statistics in high dimensions: (a) for $s \gg 1$ ($u_0 = 0$) and (b) for $s \sim 1$ ($u_0 = 22.3$). The choice of parameters $k_1 = 0.0652$, $B = 0.230$ corresponds to numerical data for $d = 4$ [11].

Dependence $y(x)$ in parametric form is given by the equations

$$y = \frac{\sigma^2}{\sigma_P^2} = k_1 u \ln\frac{1 + k_1 + k_1 u}{k_1 + k_1 u}, \quad \pm x^2 = \frac{(1+u)^{1/2} - B(u - u_0)}{(u - u_0)^{1/2}}, \quad (14)$$

where the running variable u changes from u_0 to infinity. Parameters B and k_1 are chosen according to the procedure described in [11]; parameter u_0 accounts for the finiteness of s and disappears for $s \longrightarrow \infty$. The form of Eqs. (14) is the same for all $d \geq 4$, while the choice of parameters depends on d.

In practice, the following quantity is used as a scaling variable [14]:

$$J_0 = \frac{1}{2}\langle s^2 \rangle = \frac{1}{2}\int_0^\infty s^2 P(s) ds \quad (15)$$

or another quantity η [15], closely related to it:

$$\eta = \frac{J_0 - J_{0W}}{J_{0P} - J_{0W}} = \frac{J_0 - 0.643}{0.357}, \quad (16)$$

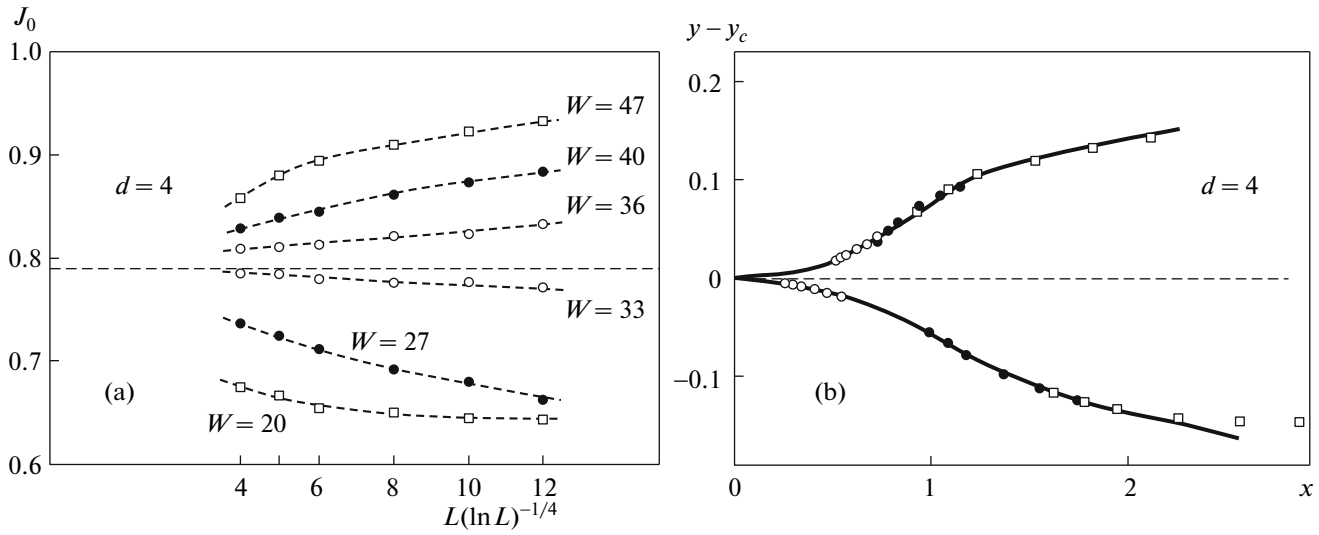

Fig. 6. (a) Numerical data by Zharekeshev and Kramer for $d = 4$ (level statistics) extracted from Fig. 4 of [14] and represented as a functional dependence on the modified length $J_0(L) = L(\ln L)^{-1/4}$; (b) their comparison with the theoretical scaling dependence (Fig. 5b).

where indices W and P denote the values of J_0 for the Wigner–Dyson and Poisson statistics, $J_{0W} = 0.643$, $J_{0P} = 1$.

Quantities J_0 and η are regular functions of the variable (10) for some $s \sim 1$ [11], so their small deviations from the critical values are proportional to each other:

$$J_0 - J_{0c} \sim \eta - \eta_c \sim y - y_c. \tag{17}$$

Examples of dependences $y(x)$ for $s \longrightarrow \infty$ (a) and $s \sim 1$ (b) are presented in Fig. 5; they correspond to the critical value $A_c = 1.4$ of parameter A in (11), which is specific for $d = 4$ [14]. Parameter u_0 in Fig. 5b is chosen so as to provide the approximate symmetry of two branches, discovered in numerical experiments by Zharekeshev and Kramer [14] (Fig. 6a). These data can be successfully matched with the theoretical dependence (Fig. 6b).

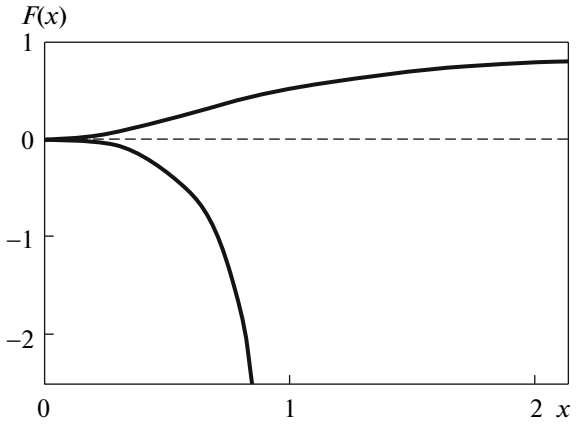

Fig. 7. Universal scaling dependence for level statistics corresponding to high dimensions.

The critical value y_c (Fig. 5a) tends to unity with the increase of d. Indeed, starting from the critical values $A_c = 1.4$ ($d = 4$) [14], $A_c = 1.17$ ($d = 5$) [15], $A_c = 1.13$ ($d = 6$) [15] and following the procedure in [11], one can obtain $y_c = 0.714$ ($d = 4$), $y_c = 0.858$ ($d = 5$), $y_c = 0.885$ ($d = 6$). This tendency agrees with the theorem [35, 36] that level statistics for the Bete lattice (corresponding to $d = \infty$) has the Poisson form even in the metallic phase.

With this observation, it is possible to obtain the universal scaling function for high dimensions. Assuming $1 - y \ll 1$, one can expand the first equation (14) in $1/k_1 u$ and linearize the right-hand side of the second equation (14) near the critical value u_c. Then

$$y - y_c = \text{const} F(x), \quad F(x) = \frac{\pm x^2}{1 \pm x^2}, \tag{18}$$

where the appropriate choice of the common scale along the x axis is made; signs + and − correspond to the upper and lower branches of function $F(x)$ shown in Fig. 7; singularity at $x = 1$ is fictitious and lies beyond the limits of applicability of (18). According to (17), deviations of J_0 and η from the critical values are described by the same function.

Figures 8a and 9a show numerical data by Garcia-Garcia and Cuevas [15] for $d = 5$ and $d = 6$, represented as a functional dependence on the modified length $\mu(L) = L^{d/4}$; they are in a good agreement with the universal scaling function $F(x)$ (see Figs. 8b, 9b).

In the cases of $d = 5$ and $d = 6$ (as opposed to $d = 4$), corrections related to the finiteness of s are insignificant. Indeed, according to [14], the quantity J_0 at $d = 4$ runs the interval (0.64, 0.79) along the lower

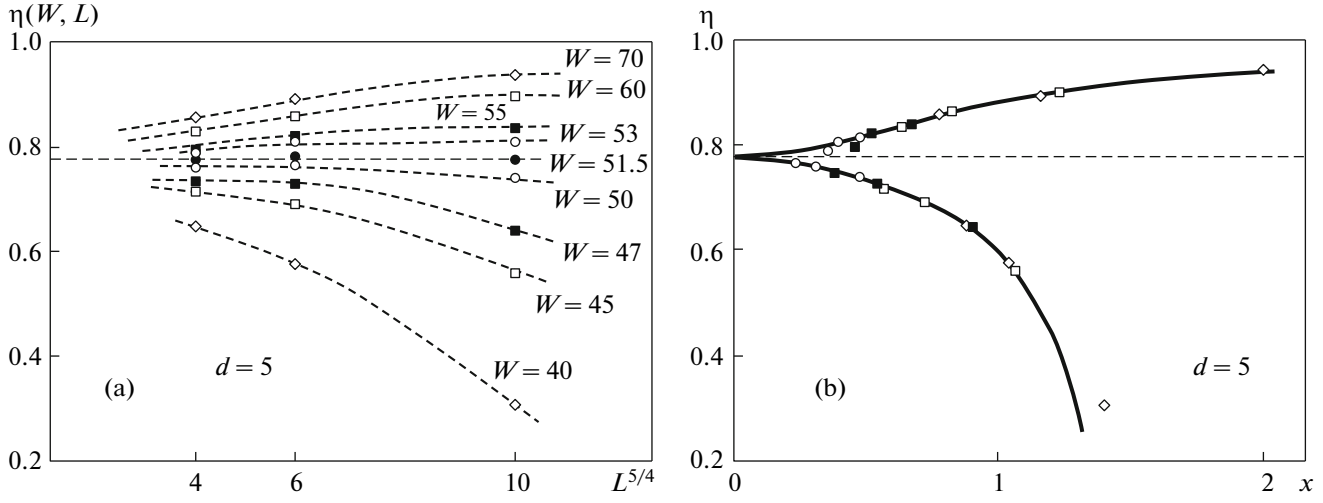

Fig. 8. Numerical data by Garcia-Garcia and Cuevas for $d = 5$ (level statistics) extracted from Fig. 1 of [15] (a); their comparison with the theoretical scaling dependence shown in Fig. 7 (b).

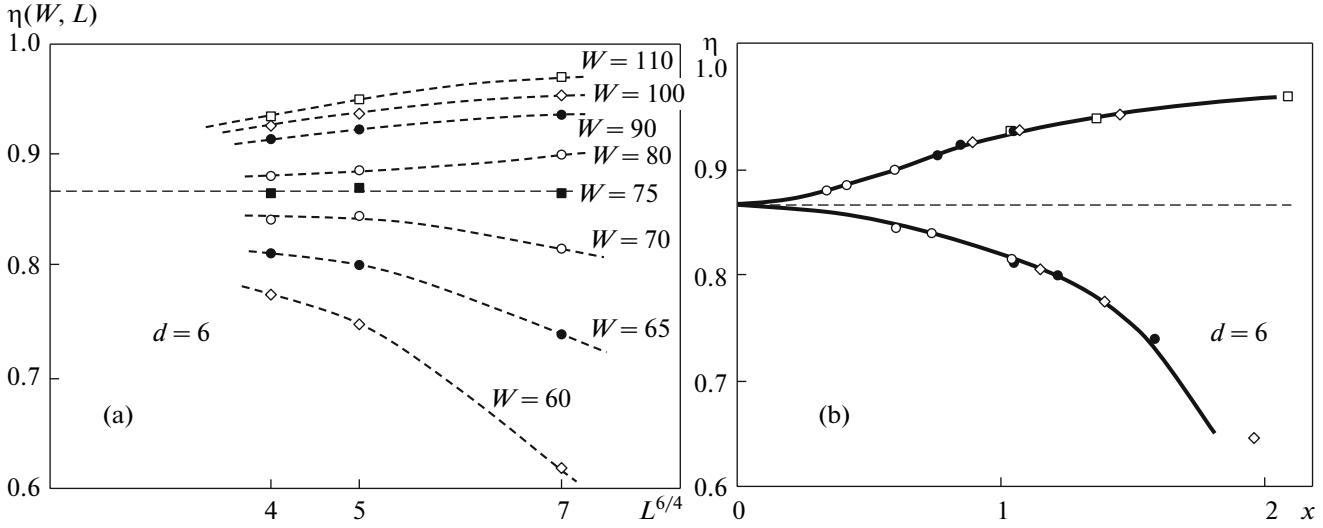

Fig. 9. Numerical data by Garcia-Garcia and Cuevas for $d = 6$ (level statistics) extracted from Fig. 1 of [15] (a); their comparison with the universal scaling dependence (b).

branch, and the interval (0.79, 1.00), along the upper branch. According to [15], these intervals are (0.64, 0.92), (0.92, 1.00) for $d = 5$, and (0.64, 0.95), (0.95, 1.00) for $d = 6$. If the choice of u_0 gives the same proportion for the intervals (y_m, y_c) and $(y_c, 1)$ in Fig. 5b, then $1 - y_c \approx y_c - y_m$ for $d = 4$ and $1 - y_c \ll y_c - y_m$ for $d = 5, 6$. In the latter case, the difference of y_m from zero is practically not manifested in the region of applicability for (18).

For $d > 4$, Eqs. (14) define scaling of the form

$$y = \frac{\sigma^2}{\sigma_P^2} = F\left(\frac{L^{d/4}}{\xi a^{(d-4)/4}}\right), \qquad (19)$$

which for large ξ gives

$$y = \tilde{F}\left(\tau \frac{L^{d/4\nu}}{a^{(d-4)/4\nu}}\right) = y_c + C\tau L^{d/4\nu} + \ldots, \qquad (20)$$

i.e., derivative y'_τ at $\tau = 0$ has a behavior of $L^{d/4\nu}$ instead of $L^{1/\nu}$, corresponding to scaling of type (1). Hence, the values of exponents $\nu = 0.84$ ($d = 5$), $\nu = 0.78$ ($d = 6$) obtained in [15] using (1) transform to $\nu = 0.67$ ($d = 5$), $\nu = 0.52$ ($d = 6$) if (19) is used. Therefore, the results of [15] for $d = 5, 6$ become close to $\nu = 1/2$ simply as a result of transfer to a correct scaling relation. The situation for $d = 4$ is analogous to that in Section 3; i.e., the main body of data corresponds to quasi-linear portions $y \sim x \sim L(\ln L)^{-1/4}$ of the scaling

curves, which was interpreted in [14, 15] as $L^{1/\nu}$ with $\nu \approx 1$. Such a situation is aggravated by the adopted treatment scheme when the derivative over $\tau = W - W_c$ is determined by expansion over $W - W_c$ and fitting by a polynomial of finite degree. In such a procedure, the result is dominated by experimental points remote from W_c, and linearity in x is retained even in the case when data close to W_c demonstrate significant nonlinearity.

5. CONCLUSIONS

The paper proposes a new interpretation of existing numerical data for the Anderson transition in high dimensions: results for $d = 4$, 5 obtained from scaling in quasi-one-dimensional systems and the results for $d = 4$, 5, 6 obtained from level statistics. Such reinterpretation is necessary due to the absence of one-parameter scaling [9] in high dimensions, which is a consequence of nonrenormalizability of the theory. All indicated numerical data appear compatible with the theoretical scaling dependences obtained from Vollhardt and Wölfle's self-consistent theory of localization. It supports the same tendency observed in [10, 11, 20, 21]: on the level of raw data, Vollhardt and Wölfle's theory appears satisfactory, while the opposite statements of the original papers are related to the ambiguity of the treatment procedure. It gives new arguments in favor of the viewpoint [18, 19] that Vollhardt and Wölfle's theory predicts the exact critical behavior.

REFERENCES

1. N. N. Bogoliubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields* (Nauka, Moscow, 1976; Wiley, New York, 1980).
2. E. Brezin, J. C. Le Guillou, and J. Zinn-Justin, in *Phase Transitions and Critical Phenomena*, Ed. by C. Domb and M. S. Green (Academic, New York, 1976), Vol. VI.
3. S. Ma, *Modern Theory of Critical Phenomena* (Benjamin, Reading, Massachusetts, United States, 1976).
4. A. Nitzan, K. F. Freed, and M. N. Cohen, Phys. Rev. **15**, 4476 (1977).
5. M. V. Sadovskii, Sov. Phys.—Usp. **24**, 96 (1981)
6. I. M. Suslov, Phys—Usp. **41**, 441 (1998).
7. J. J. M. Verbaarschot and M. R. Zirnbauer, J. Phys. A: Math. Gen. **18**, 1093 (1985).
8. M. R. Zirnbauer, cond-mat/9903338.
9. E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishman, Phys. Rev. Lett. **42**, 673 (1979).
10. I. M. Suslov, J. Exp. Theor. Phys **114** (1), 107 (2012).
11. I. M. Suslov, J. Exp. Theor. Phys. **118** (6), 909 (2014); I. M. Suslov, arXiv:1402.2382.
12. P. Markos, Acta Phys. Slovaca **56**, 561 (2006); P. Markos, cond-mat/0609580.
13. P. Marko, J. Exp. Theor. Phys. **115** (6), 1075 (2012).
14. I. Kh. Zharekeshev and B. Kramer, Ann. Phys. (Leipzig) **7**, 442 (1998).
15. A. M. Garcia-Garcia and E. Cuevas, Phys. Rev. B: Condens. Matter **75**, 174203 (2007).
16. F. Wegner, Z. Phys. B: Condens. Matter **35**, 207 (1979); L. Schäfer and F. Wegner, Z. Phys. B: Condens. Matter **38**, 113 (1980). S. Hikami, Phys. Rev. B: Condens. Matter **24**, 2671 (1981). K. B. Efetov, A. I. Larkin, and D. E. Khmelnitskii, Sov. Phys. JETP **52** (3), 568 (1980); K. B. Efetov, Adv. Phys. **32**, 53 (1983).
17. D. Vollhardt and P. Wölfle, Phys. Rev. B: Condens. Matter **22**, 4666 (1980); D. Vollhardt and P. Wölfle, Phys. Rev. Lett. **48**, 699 (1982); D. Vollhardt and P. Wölfle, in *Modern Problems in Condensed Matter Sciences*, Ed. by V. M. Agranovich and A. A. Maradudin (North-Holland, Amsterdam, The Netherlands, 1992), Vol. 32.
18. H. Kunz and R. Souillard, J. Phys., Lett. **44**, L506 (1983).
19. I. M. Suslov, J. Exp. Theor. Phys **81** (5), 925 (1995).
20. I. M. Suslov, J. Exp. Theor. Phys **115** (5), 897 (2012).
21. I. M. Suslov, J. Exp. Theor. Phys **115** (6), 1079 (2012).
22. K. B. Efetov, Sov. Phys. JETP **61** (3), 606 (1985).
23. K. B. Efetov, Sov. Phys. JETP **66** (3), 634 (1987); K. B. Efetov, Sov. Phys. JETP **67** (1), 199 (1988).
24. M. R. Zirnbauer, Phys. Rev. B: Condens. Matter **34,** 6394 (1986); M. R. Zirnbauer, Nucl. Phys. B **265**, 375 (1986).
25. A. D. Mirlin and Y. V. Fyodorov, Phys. Rev. Lett. **72**, 526 (1994).
26. F. Evers and A. D. Mirlin, Rev. Mod. Phys. **80**, 1355 (2008).
27. A. M. Garcia-Garcia, Phys. Rev. Lett. **100**, 076404 (2008).
28. K. B. Efetov, Sov. Phys. JETP **56** (2), 467 (1982).
29. M. Moshe and J. Zinn-Justin, Phys. Rep. **385**, 69 (2003).
30. B. Shapiro, Phys. Rev. Lett. **50**, 747 (1983).
31. H. Kunz and R. Souillard, J. Phys., Lett. **44**, L411 (1983).
32. J. T. Chalker, Physica A (Amsterdam) **167**, 253 (1990); T. Brandes, B. Huckestein, and L. Schweitzer, Ann. Phys. (Leipzig) **5**, 633 (1996).
33. I. M. Suslov, cond-mat/0612654.
34. J. L. Pichard and G. Sarma, J. Phys. C: Solid State Phys. **14**, L127 (1981).
35. A. MacKinnon and B. Kramer, Phys. Rev. Lett. **47**, 1546 (1981).
36. M. Aizenman and S. Warzel, Math. Phys. Anal. Geom. **9**, 291 (2006).